Universal Nonlinear Features of Intra-Day Price Dynamics

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Abstract

How and why stock prices move is a centuries-old question still not answered conclusively. Here we reveal that price impact has universal non-linear features for trades aggregated on any intra-day scale. We find that the classification of trades as price-changing versus non-price-changing can explain many of the aforementioned features of the price impact. We also show that some propagator models can reproduce these effects.
Our findings challenge the widespread assumption of linear aggregate impact. They imply that market dynamics on all intra-day timescales are shaped by correlations and bilateral adaptation in the flows of liquidity provision and taking.

See our papers and code referenced below for more details and solutions to several long-standing technical issues.

Transient, History-Dependent, and Constant Impact Models

We model returns as a convolution of market order signs ϵ with kernels g or κ,

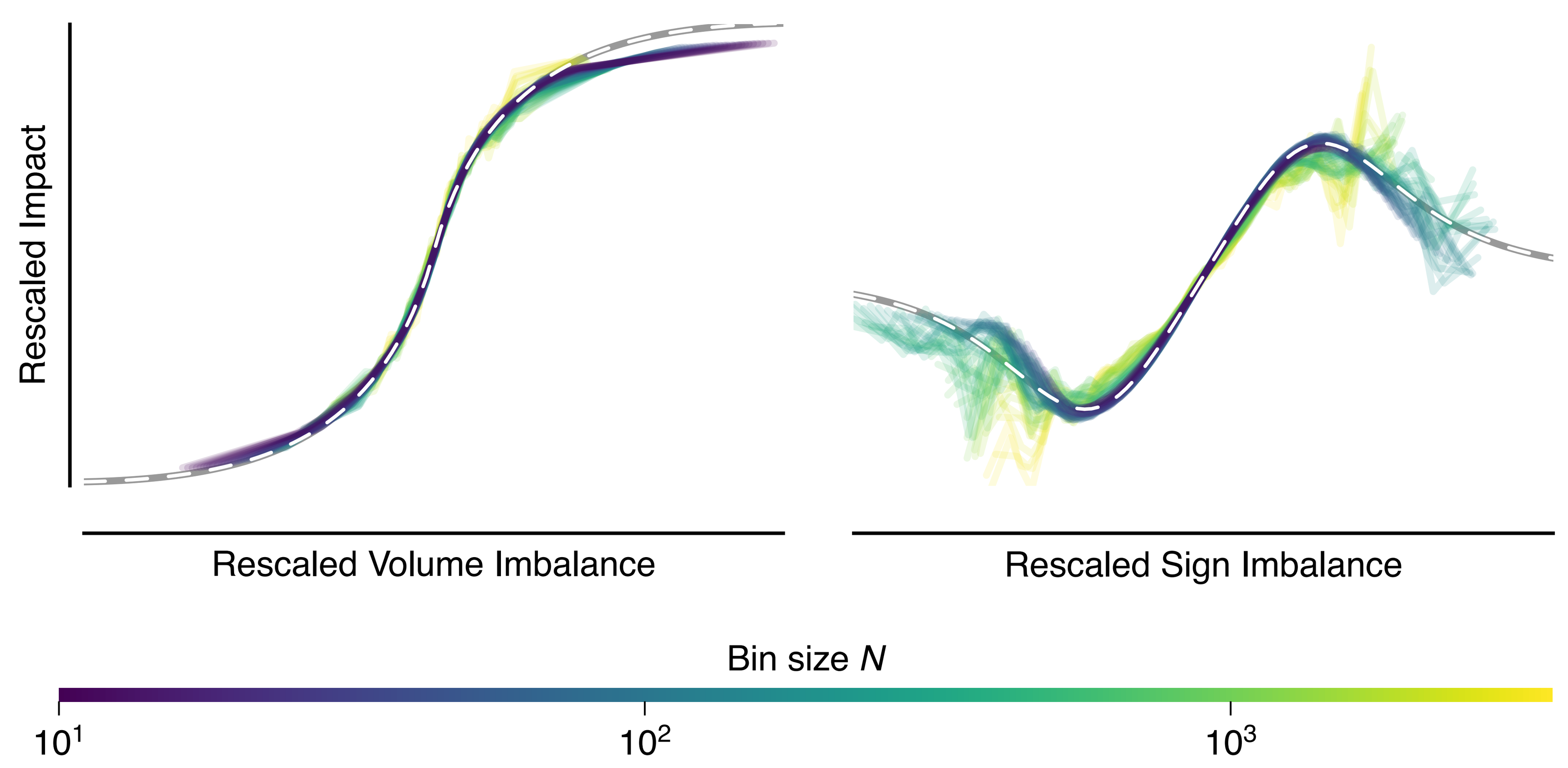
$$r_{\mathrm{TIM1}}(t) := \sum_{j\geq 0} g(j)\,\epsilon(t-j) \qquad r_{\mathrm{TIM2}}(t) := \sum_{\pi'}\sum_{j\geq 0} g_{\pi'}(j)\,\delta_{\pi(t-j)\,\pi'}\,\epsilon(t-j)$$

$$r_{\mathrm{HDIM2}}(t) := \sum_{\pi''} \delta_{\pi(t)\,\pi''} \sum_{\pi'}\sum_{j\geq 0} \kappa_{\pi'\pi''}(j)\,\delta_{\pi(t-j)\,\pi'}\,\epsilon(t-j) \qquad r_{\mathrm{CIM2}}(t) := \Delta_c\,\delta_{\pi(t)\,c}\,\epsilon(t)$$

Except for TIM1, kernels depend on labels π indicating changes of the mid-price:

$$\pi(t) = \begin{cases} n & \text{if} \quad m(t+1) = m(t) \\ c & \text{else} \end{cases}$$

Nonlinear Impact Master Curves (example: AAPL 2016)



We measure the impact of N trades as

$$\mathcal{R}_N(X) := \left\langle \log m(t+N) - \log m(t) \,\middle|\, X = \sum_{i=0}^{N-1} x_{t+i} \right\rangle$$

where m is the mid price, $x = q$, the signed volume of single trade, or $x = \epsilon$, the sign of the market order. The corresponding aggregate quantities are represented with capital letters.

Master curve (--) and rescaling:

$$\mathcal{R}_N(\mathcal{Q}) \approx R_N\,\mathcal{F}\left(\frac{\mathcal{Q}}{Q_N}\right)$$

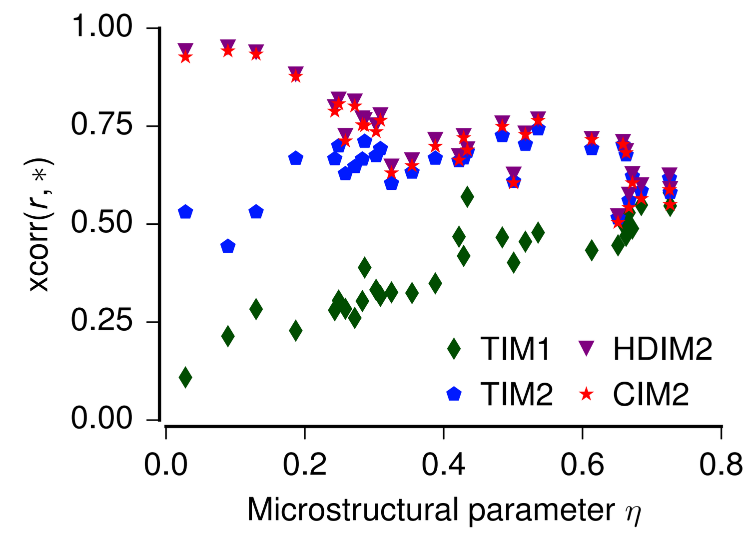
$$Q_N \approx Q_1 N^{\xi}$$

$$R_N \approx R_1 N^{\psi}$$

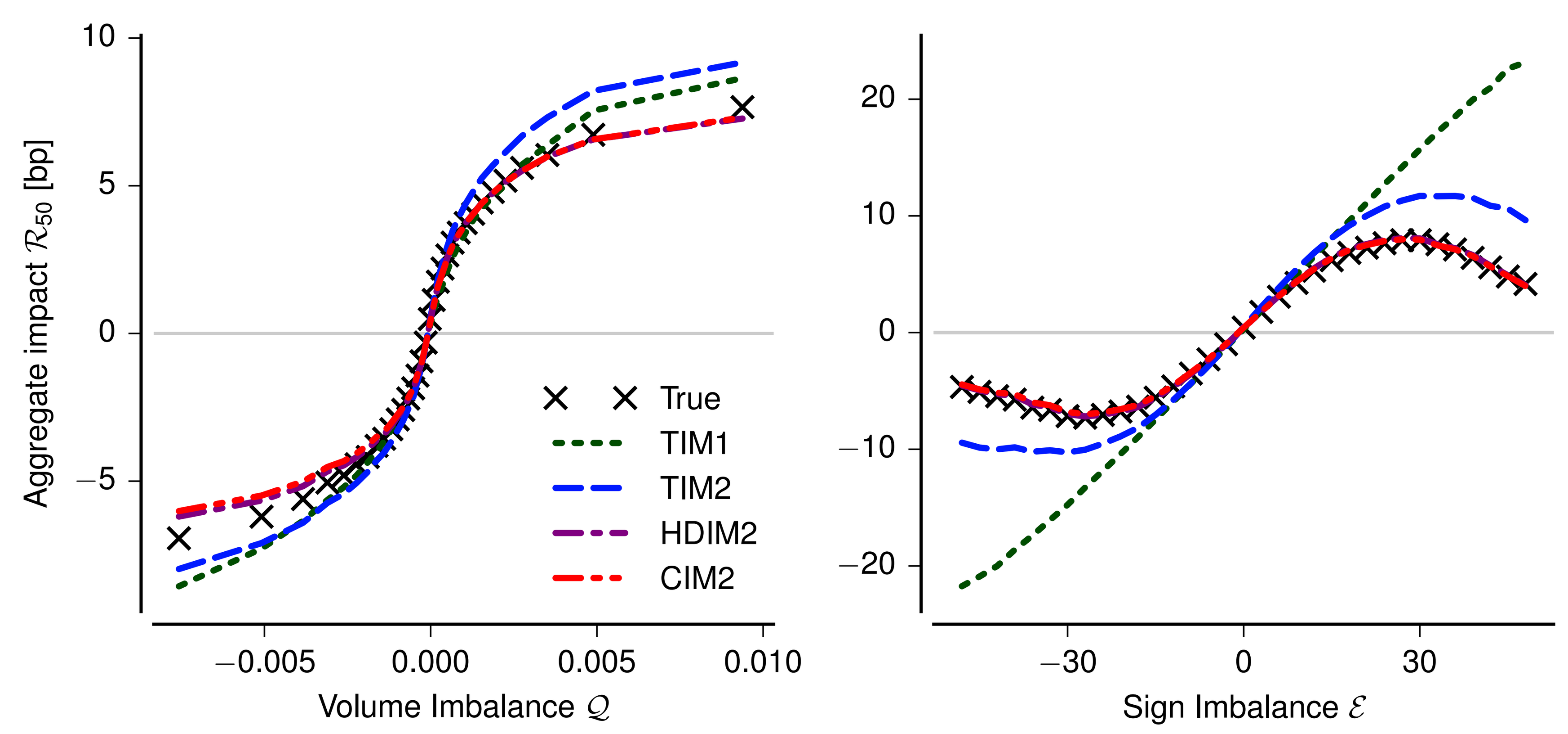
$$\mathcal{F}(x) = \frac{x}{(1+|x|^{\alpha})^{\beta/\alpha}},$$

Time Series Cross Correlations

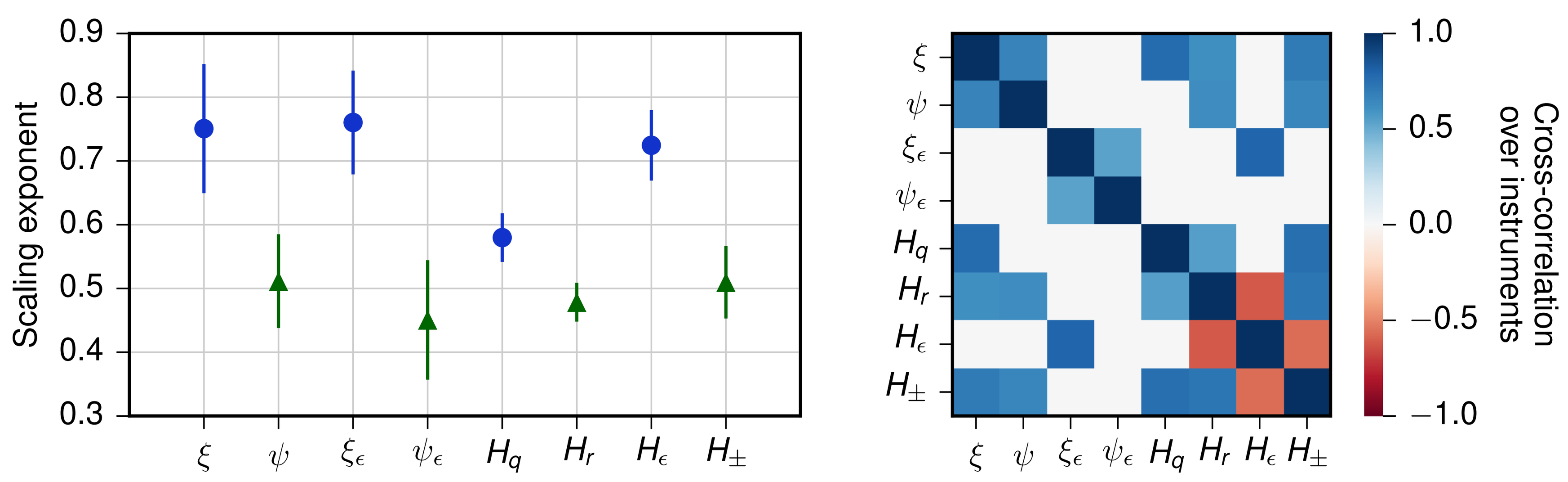
...between real and predicted returns reveal that model performance depends on the tick size.
η measures the effect of price discretisation ($\eta < 0.5$: small tick, $\eta > 0.5$: large tick).



Master Curve Shapes (example : MSFT 2015-2016)

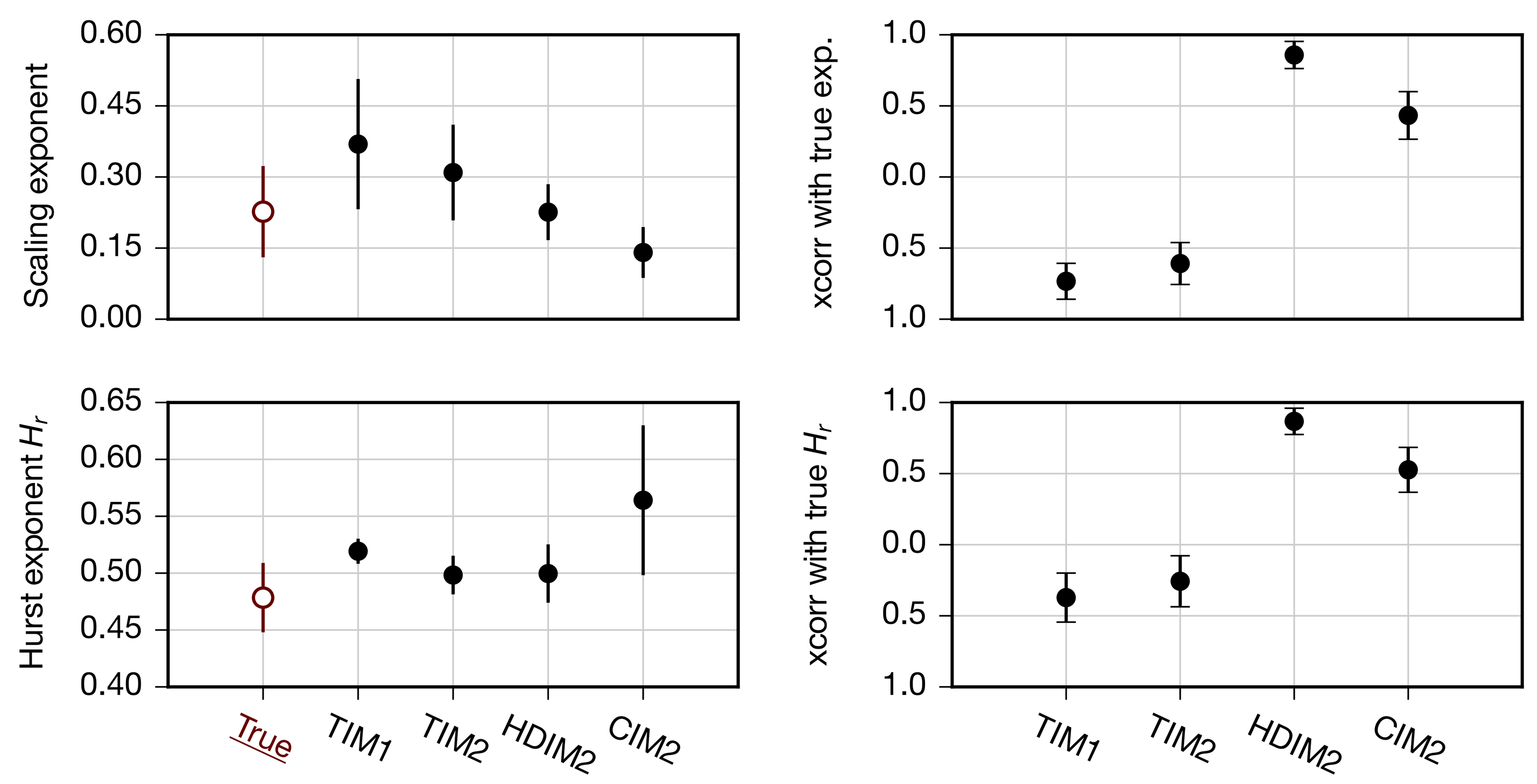


Impact Rescaling Exponents Are Similar to the Corresponding Hurst Exponents

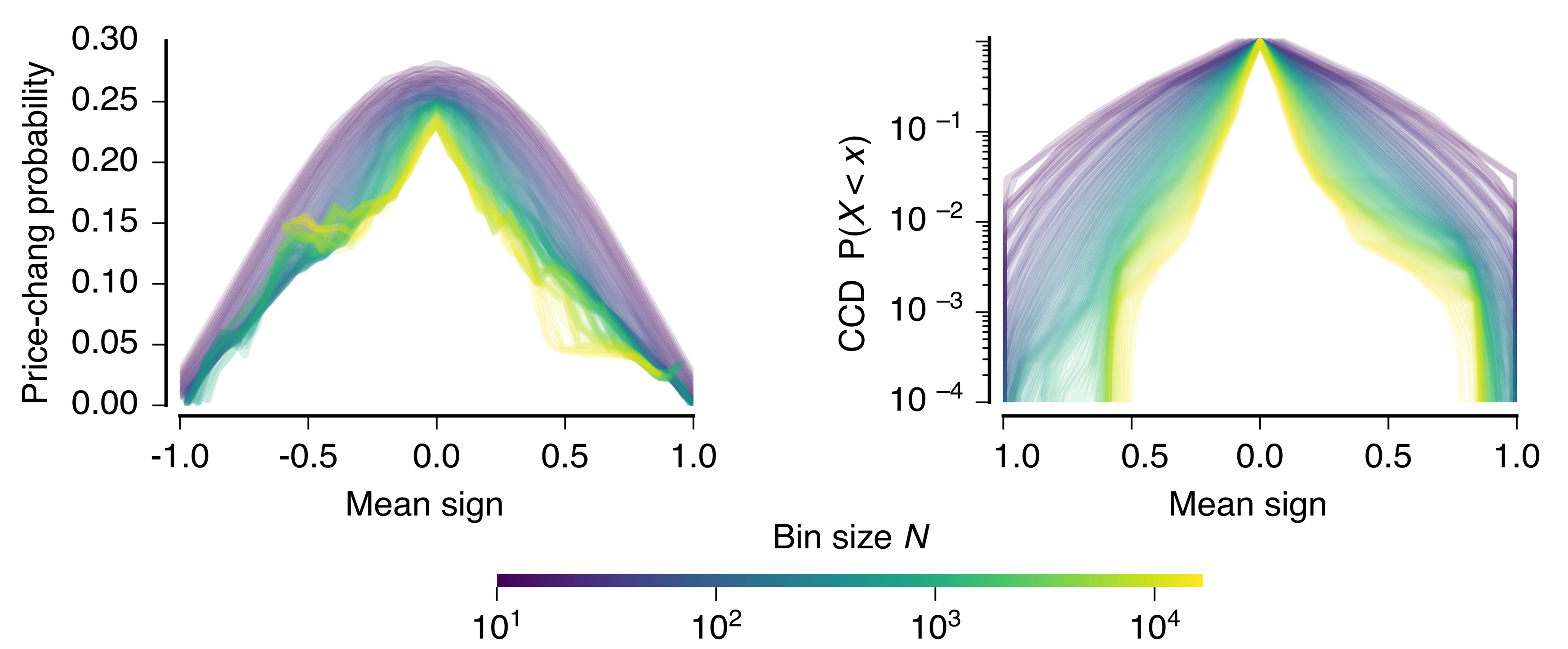


Impact curve x- and y- axis rescaling exponents: ξ and ψ for aggregate volume, ξ_ϵ, ψ_ϵ for aggregate signs. Hurst exponents: H_q for signed volume, H_r for returns, H_ϵ for market order signs, $H_\pm$ for return signs.

Impact Slope Scaling (ξ-ψ) and Hurst Exponent.



Universal link between order-flow bias and price "stickyness" (example: EUROSTOXX 2015)



The Data

- 12 on stocks NASDAQ (primary market), for 2011-2016
- 13 highest turnover stocks on OMX for 2011-2015.
- 6 futures on EUREX EBS 2014-2015

Transactions with the same sign and millisecond-timestamp were merged into one estimated trade.

This yields 10^3 – 10^4 trades per instrument per day

See Also

Patzelt & Bouchaud (2017): Universal scaling and nonlinearity of aggregate price impact in financial markets. In review. arXiv:1706.04163

Patzelt & Bouchaud (2017): Nonlinear price impact from linear models. In press (JSTAT). arXiv:1708.02411

github.com/felixpatzelt/priceprop